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**McGill University**
**ECON 763**
**Financial econometrics**
**Final exam**

No documentation allowed
Time allowed: 3 hours

20 points 1. Consider the following ARMA model:

$$X_t = 10 + u_t - 0.75\, u_{t-1} + 0.125\, u_{t-2} \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0,1)$ sequence. Answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

  i. $E(X_t)$;
  ii. $\gamma(k)$, $k = 1, \ldots, 8$;
  iii. $\rho(k)$, $k = 1, 2, \ldots, 8$.

(d) Graph $\rho(k)$, $k = 1, 2, \ldots, 8$.

(e) Find the coefficients of $u_t$, $u_{t-1}$, $u_{t-2}$, $u_{t-3}$ and $u_{t-4}$ in the moving average representation of $X_t$.

(f) Compute the first two partial autocorrelations of $X_t$.

(g) If $X_{10} = 1$ and assuming the parameters of the model are known, can you compute the best linear forecasts of $X_{10}$, $X_{11}$, $X_{12}$ and $X_{13}$ based on $X_{10}$ (only)? If so, compute these.

(h) If $X_{10} = 1$, $u_{10} = 2$, $u_9 = 1$, $u_8 = 0.99$, $u_7 = 1.2$, and assuming the parameters of the model are known, can you compute the best linear forecasts of $X_{11}$, $X_{12}$ and $X_{13}$ based on the history of the process up to $X_{10}$? If so, compute these.

20 points **2.** Let $X_1, X_2, \ldots, X_T$ be a time series.

(a) Define:

i. the sample autocorrelations for this series;

ii. the partial autocorrelations for this series.

(b) Discuss the asymptotic distributions of these two sets of autocorrelations in the following cases:

i. under the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.);

ii. under the hypothesis that the process follows a moving average of finite order.

(c) Describe how you would identify the process described in equation (1) in question 1.

(d) Propose a method for testing the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.) without any assumption on the existence of moments for $X_1, X_2, \ldots, X_T$. In particular, discuss:

i. how bounds could be applied to test this type of hypothesis;

ii. how a simulation-based procedure could be used.

20 points **3.** Let $X_1, X_2, \ldots, X_T$ be a time series where $X_1, X_2, \ldots, X_T$ have continuous distributions.

(a) Propose a method for testing the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.) without any assumption on the existence of the moments for $X_1, X_2, \ldots, X_T$.

(b) If $X_1, X_2, \ldots, X_T$ have common median $m_0$, describe a procedure for testing whether these observations are independent without assuming identical distributions.

(c) Consider the "median regression" model:

$$y_t = x_t'\beta + u_t,\ t = 1, \ldots, T, \tag{2}$$

where $x_t$, $t = 1, \ldots, T$, are $k \times 1$ fixed vectors and the the disturbances $u_t$, $t = 1, \ldots, T$, are independent with median zero and continuous distributions. Propose procedures for testing hypotheses of the form $H_0 : \beta = \beta_0$ and build confidence sets for $\beta$.

(d) Discuss how the procedure described could be adapted if the errors in (2) have discrete distributions.

20 points

4. Let $R_{it}$, $i = 1, \ldots, n$, be returns on $n$ securities for period $t$, and $\tilde{R}_{\mathrm{M}t}$ the return on a benchmark portfolio $(t = 1, \ldots, T)$. The (unconditional) CAPM which assumes time-invariant *betas* can be assessed by testing:

$$\mathscr{H}_{\mathrm{E}} : a_i = 0\,, \quad i = 1, \ldots, n, \tag{3}$$

in the context of the MLR model

$$r_{it} = a_i + \beta_i \tilde{r}_{\mathrm{M}t} + \varepsilon_{it}\,, \quad t = 1, \ldots, T,\ i = 1, \ldots, n, \tag{4}$$

where $r_{it} = R_{it} - R_{ft}$, $\tilde{r}_{\mathrm{M}t} = \tilde{R}_{\mathrm{M}t} - R_{ft}$, $R_{ft}$ is the riskless rate of return and $\varepsilon_{it}$ is a random disturbance, such that

$$V_t \equiv (\varepsilon_{1t}, \ldots, \varepsilon_{nt})' = JW_t\,,\ t = 1, \ldots,\ T\,, \tag{5}$$

where $J$ is an unknown, non-singular matrix and the distribution of the vector $w = vec\,(W)$, $W = [W_1, \ldots, W_T]'$ is either: (i) known (hence, free of nuisance parameters), or (ii) specified up to an unknown finite dimensional nuisance-parameter (denoted $\nu$).

(a) Put the model (4) in matrix notation.

(b) On assuming that the vectors $W_1, \ldots, W_T$ are i.i.d. $N[0, I_n]$, describe the likelihood ratio test for $\mathscr{H}_E$, and discuss how this test could be implemented.

(c) Propose a procedure for testing whether the errors $W_1, \ldots, W_T$ are i.i.d. $N[0, I_n]$.

(d) If another distribution is assumed for $w$ (such as a heavy-tailed distribution), discuss how such a test could be implemented.

20 points

5. Consider a time series of asset returns $R_t$, $t = 1, \ldots, T$, which are i.i.d. according to stable distribution, with characteristic function

$$\ln \int_{-\infty}^{\infty} e^{ist} d\mathrm{P}(S < s) = \begin{cases} -\sigma^{\alpha}|t|^{\alpha}[1 - i\beta\ \mathrm{sign}(t)\tan\frac{\pi\alpha}{2}] + i\mu t, & \text{for } \alpha \neq 1, \\ -\sigma|t|[1 + i\beta\frac{\pi}{2}\ \mathrm{sign}(t)\ln|t|] + i\mu t, & \text{for } \alpha = 1. \end{cases} \tag{6}$$

(a) Discuss the interpretation of the different parameters $\mu$, $\sigma$, $\alpha$ and $\beta$.

(b) Why are stable random variables called "stable"?

(c) On assuming that $\beta = 0$, propose a method for testing

$$H_0(\alpha_0) : \alpha = \alpha_0. \tag{7}$$

(d) On assuming that $\beta = 0$, discuss how a confidence set for $\alpha$ could be built.