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McGill University
ECN 706
Special topics in econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

20 points 1. Describe the main statistical problems as decision problems.

(a) Explain the difference between a *nonrandomized* decision rule and a *randomized* decision rule.

(b) Define the risk function for each one of these two types of rule.

(c) When is a decision rule *admissible*?

20 points 2. Let $\ell(Y;\theta)$ be the likelihood function for the sample $Y = (Y_1, \ldots, Y_n)'$. Show that

$$I(\theta) = E\left[-\frac{\partial^2 \log \ell(Y;\theta)}{\partial\theta\,\partial\theta'}\right] .$$

30 points 3. Consider the linear regression model

$$y = X\beta + u \tag{0.1}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ fixed matrix of explanatory variables (observed), $\beta = (\beta_1, \ldots, \beta_k)'$, and $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of unobserved error terms. Suppose the elements of u are independent and identically distributed according to a $\sigma t(1)$ distribution, where $t(1)$ represents a Student t distribution with 1 degree of freedom and σ is an unknown constant.

(a) Propose a method for testing the hypothesis $H_0 : \beta_1 = 1$ at level $\alpha = 0.05$ in the context of this model such that the size of the test is exactly equal to $\alpha = 0.05$.

(b) Propose a test for detecting serial dependence between the errors $u_1, \ldots, u_T$ such the size of the test is exactly equal to $\alpha = 0.05$.

30 points 4. Consider the linear regression model

$$y = X\beta + u \tag{0.2}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ fixed matrix of explanatory variables (observed), $\beta = (\beta_1, \ldots, \beta_k)'$, and $u = (u_1, \ldots, u_T)'$ is a $T \times 1$ vector of unobserved error terms, and $k \geq 3$. We wish to test the hypothesis

$$H_0 : \beta_2\beta_3 = 1. \tag{0.3}$$

(a) If $u \sim \mathrm{N}[0, \sigma^2 I_T]$, describe the likelihood ratio (LR) criterion test for testing H_0 against the unrestricted model.

(b) Propose a bound for the null distribution the above LR statistic.

(c) Can you suggest a simulation-based procedure which could eventually improve the above bound?