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# ECONOMETRIC THEORY
# REVIEW QUESTIONS

## Weak identification

1. Provide brief answers to the following questions (maximum of 1 page per question).

   (a) Explain the difference between the "level" of a test and its "size".

   (b) Explain the difference between the "level" of a confidence set and its "size".

   (c) Discuss the link between tests and confidence sets: how confidence sets can be derived from tests, and vice-versa.

2. Provide brief answers to the following questions (maximum of 1 page per question).

   (a) Explain what the Bahadur-Savage theorem entails for testing in nonparametric models.

   (b) Suppose we wish to test the hypothesis

   $$H_0 : \quad X_1, \ldots, X_n \text{ are independent random variables each with a distribution symmetric about zero.} \tag{1}$$

   What condition should this test satisfy to have level $0.05$.

3. Provide brief answers to the following questions (maximum of 1 page per question).

   (a) Explain the notion of weak identification.

   (b) Discuss the consequences of the possible lack of identification on the construction of confidence sets.

   (c) Explain the notion of "identification-robust" method.

   (d) In the context of a linear simultaneous equations model, provide an example of a method which is identification-robust and a method which is not identification-robust.

4. Consider the following simultaneous equations model:

$$y = Y\beta + X_1\gamma + u\,, \tag{2}$$
$$Y = X_1\mathit{\Pi}_1 + X_2\mathit{\Pi}_2 + V\,, \tag{3}$$

where $y$ and $Y$ are $T \times 1$ and $T \times G$ matrices of endogenous variables, $X_1$ and $X_2$ are $T \times k_1$ and $T \times k_2$ matrices of exogenous variables, $\beta$ and $\gamma$ are $G \times 1$ and $k_1 \times 1$ vectors of unknown coefficients, $\mathit{\Pi}_1$ and $\mathit{\Pi}_2$ are $k_1 \times G$ and $k_2 \times G$ matrices of unknown coefficients, $u = (u_1,\, \ldots\, ,\, u_T)'$ is a $T \times 1$ vector of random disturbances, $V = [V_1,\, \ldots\, ,\, V_T]'$ is a $T \times G$ matrix of random disturbances,

$$X = [X_1,\, X_2] \text{ is a } T \times k \text{ full-column rank matrix,} \tag{4}$$

where $k = k_1 + k_2$, and

$$u \text{ and } X \text{ are independent,} \tag{5}$$
$$u \sim N\big[0,\, \sigma_u^2\, I_T\big]\,. \tag{6}$$

(a) Discuss the conditions under which the parameters of equation (2) are identified;

(b) if $G = 1$, propose an exact confidence region for $\beta$;

(c) if $G \geq 2$, propose an exact confidence region for $\beta$;

(d) if $G \geq 2$, propose an exact confidence region for each component of $\beta$;

(e) describe an exact procedure for testing an hypothesis of the form:

$$H_0 : \beta = \beta_0 \text{ and } \gamma = \gamma_0 \tag{7}$$

where $\beta_0$ and $\gamma_0$ are given values;

(f) propose an exact confidence region for $\gamma$.

# References


DUFOUR, J.-M. (2003): "Identification, Weak Instruments and Statistical Inference in Econometrics," *Canadian Journal of Economics*, 36(4), 767–808.