ECONOMETRIC THEORY
REVIEW QUESTIONS
Multivariate dynamic models

1. Discuss the relationships between the following concepts:
   (a) Granger causality and prediction;
   (b) Granger causality and causality at several horizons;
   (c) Granger causality and impulse response coefficients;
   (d) causality at several horizons and impulse response coefficients.

2. Consider the process described by the following model:

   \[ X_t = \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} = \begin{bmatrix} 1 - 0.5B \\ -0.5B \end{bmatrix} \begin{bmatrix} 0 \\ 1 - 0.2B \end{bmatrix} \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \]

   où \( t \in \mathbb{Z} \), \( a_t = [a_{1t}, a_{2t}]' \) is a sequence of \( i.i.d. \) \( N[0, \Sigma] \) random vectors with

   \[ \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \]

   (a) What is the type of this process?
   (b) Is this process strictly stationary? Why?
   (c) Does this process have a Wold representation? If so, give it.
   (d) Is this process invertible? Why?
   (e) Does this process has an autoregressive representation? If so, give it.
   (f) Does the variable \( X_{2t} \) cause \( X_{1t} \) in the sense of Granger? Justify your answer.
   (g) Does the variable \( X_{1t} \) cause \( X_{2t} \) in the sense of Granger? Justify your answer.
   (h) Does the variable \( X_{2t} \) cause \( X_{1t} \) at all horizons? Justify your answer.
   (i) Does the variable \( X_{1t} \) cause \( X_{2t} \) at all horizons? Justify your answer.