ECONOMETRIC THEORY
EXERCISES 1
MODELS

Reference: Gouriéroux and Monfort (1995, Chapter 1)

1. (a) Define the notion of statistical model.
   (b) Explain the distinction between a dominated statistical model and a homogeneous statistical model.
   (c) When is a model nested by another model? What is a submodel? What a nesting model?

2. (a) Explain what is an exponential statistical model.
   (b) Give two examples of exponential statistical models and explain why these models belong to the exponential family.
   (c) Is a linear model always an exponential model?
   (d) Which ones of the following terms apply to exponential models: parametric, nonparametric, semiparametric?
   (e) Which ones of the following terms apply to linear models: parametric, nonparametric, semiparametric?

3. Explain the difference between the Bayesian approach and the empirical Bayesian approach to the introduction of a priori information.

4. Let $P$ and $P^*$ be two probability distributions possessing densities with respect to the same measure $\mu$.
   (a) Define the Kullback discrepancy between $P$ and $P^*$.
   (b) Prove that:
      i. $I(P \mid P^*) \geq 0$;
      ii. $I(P \mid P^*) = 0 \iff P = P^*$.

5. Let $y = (y_1, \ldots, y_n)'$ be a vector of observations. To explain $y$, we consider the linear model:

$$y = m + u, \ m \in L, \ u \sim N \left[ 0, \sigma^2 I_n \right]$$

where $L$ is a vector $\mathbb{R}^k$. If the true probability distribution of $y$ is $N \left[ m_0, \sigma_0^2 I_n \right]$, find the pseudo true values $m_0^*, \sigma_0^*$ of $m$ and $\sigma^2$. [$I_n$ represents the identity matrix of order $n$.]
6. Consider the following simple Keynesian model:
\[ C_t = aR_t + b + u_t, \]
\[ Y_t = C_t + I_t, \]
\[ R_t = Y_t, \]
where \( C_t \) represents consumption (at time \( t \)), \( R_t \) income, \( Y_t \) production, \( I_t \) investment, and \( u_t \) is a random disturbance.

(a) Find the reduced form of this model.
(b) Is a coherency condition needed to derive this reduced form? If yes, which one and why?
(c) Does this model contain latent variables? If so, which ones?

(a) Explain the notion of exogeneity with respect to a parameter.

7. Consider the following simplified equilibrium model:
\[ D_t = \alpha + 2p_t + u_{1t}, \]
\[ S_t = c + u_{2t}, \]
\[ Q_t = D_t = S_t, \quad t = 1, \ldots, T \]
where \( D_t \) is the demand for a product, \( S_t \) the supply for the same product, and \( Q_t \) the quantity produced and sold. We suppose that the vectors \((u_{1t}, u_{2t})', t = 1, \ldots, T\) are independent and \(N[0, I_2]\).

(a) Find the reduced form of this model.
(b) For which parameters is the vector \( Q = (Q_1, \ldots, Q_T)' \) exogenous? Justify your answer.
(c) For which parameters is the vector \( p = (p_1, \ldots, p_T)' \) exogenous? Justify your answer.
(d) Are the variables \( Q_t \) and \( p_t \) simultaneous?

8. Prove the equivalence between non-causality in the sense of Granger and non-causality in the sense of Sims. (Define clearly these two notions.)

9. Give a sufficient condition under which sequential exogeneity is equivalent to exogeneity (for a parameter \( \alpha \)) and justify your answer.

10. Consider the following equilibrium model:
\[ Q_t = a + bp_t + u_{1t}, \]
\[ p_t = c + dp_{t-1} + u_{2t}, \quad t = 1, \ldots, T \]
where the disturbances \((u_{1t}, u_{2t})\), \(t = 1, \ldots, T\) are independent \(N[0, I_2]\), \(Q_t\) represents the quantity sold, and \(p_t\) the price. For which parameters is the vector \(p = (p_1, \ldots, p_T)\):

(a) sequentially exogenous?
(b) exogenous?
(c) strongly exogenous?
(d) Further, does \(Q_t\) cause \(p_t\) in the sense of Granger?

Justify your answers.

12. Consider the following equilibrium model:

\[
Q_t = a + bp_{t+1} + u_{1t}, \\
p_t = c + dQ_{t-1} + u_{2t}, \quad t = 1, \ldots, T \\
p_0 \text{ is fixed}
\]

where the disturbances \((u_{1t}, u_{2t})\), \(t = 1, \ldots, T\) are independent \(N[0, I_2]\), \(Q_t\) represents the quantity sold, and \(p_t\) the price. For which parameters is the vector \(p = (p_1, \ldots, p_T)\):

(a) exogenous for \((a, b)\)?
(b) exogenous for \((c, d)\)?
(c) sequentially exogenous for \((a, b)\)?
(d) sequentially exogenous for \((c, d)\)?
(e) strongly exogenous for \((a, b)\)?
(f) strongly exogenous for \((c, d)\)?

Justify your answers.
(c) sequentially exogenous for $(a, b)$?
(d) sequentially exogenous for $(c, d)$?
(e) strongly exogenous for $(a, b)$?
(f) strongly exogenous for $(c, d)$?

Justify your answers.

13. Consider the following equilibrium model:

\[
D_t = a + b p_t + u_{1t},
S_t = c + d p_{t-1} + e x_t + f x_{t-1} + u_{2t},
Q_t = D_t = S_t, \quad t = 1, \ldots, T
\]

where $D_t$ is the demand for a product, $S_t$ the supply for the same product, $Q_t$ the quantity produced, $x_t$ is an exogenous variable, $p_0$ and $x_0$ are fixed, and $u_t = (u_{1t}, u_{2t})'$ is random vector such that $E(u_t) = 0$.

(a) Give the structural form associated with this model.
(b) Give the reduced form of this model.
(c) Find the short-term multipliers for $p_t$ and $Q_t$.
(d) Find the final form of the model.
(e) Find the dynamic multipliers for $p_t$.
(f) Find the long-run form of the model and the long-term multipliers for $p_t$ and $Q_t$.

References